

Enrollment No: _____

Exam Seat No: _____

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name: Mathematics-I

Subject Code: 4SC01MAT1

Branch: B.Sc. (All)

Semester: 1

Date: 23/03/2018

Time: 02:30 To 05:30

Marks: 70

Instructions:

- (1) Use of Programmable calculator and any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1 Attempt the following questions:**(14)**

a) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \underline{\hspace{2cm}}$.

(01)

- (a) 1 (b) e (c) $\log 1$ (d) $\frac{1}{e}$

b) A square matrix A is called skew-symmetric matrix if _____.

(01)

- (a) $A^T = -A$ (b) $A^2 = A$ (c) $A^T = A$ (d) $A^2 = I$

c) n^{th} derivative of $y = \log(3-2x)$ is _____.

(01)

(a) $\frac{-(2)^n (n-1)!}{(3-2x)^n}$ (b) $\frac{(-1)^n (-2)^n n!}{(3-2x)^n}$ (c) $\frac{2^n n!}{(3-2x)^{n+1}}$ (d) $\frac{(-1)^{n-1} (-2)^n (n-1)!}{(3-2x)^{n+1}}$

d) The one of the Eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & 2.5 \\ 0 & \sqrt{2} & 3 \\ 0 & 0 & -1 \end{bmatrix}$ is _____.

(01)

- (a) 1 (b) 2 (c) 3 (d) 0

e) The series $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ represent expansion of _____.

(01)

- (a) $\sin x$ (b) $\cos x$ (c) $\sinh x$ (d) $\cosh x$

f) The radius of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$ is _____.

(01)

- (a) 5 (b) 2 (c) 4 (d) 6



- g) The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$ is _____. (01)
 (a) 1 (b) 2 (c) 3 (d) 0
- h) $\lim_{x \rightarrow 0} \frac{3x}{\tan 3x} =$ _____. (01)
 (a) 3 (b) $\frac{1}{3}$ (c) 1 (d) 0
- i) Formula of Lagrange's theorem is _____. (01)
 (a) $\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(c)}{g'(c)}$ (b) $\frac{f(b)-f(a)}{g(b)-g(a)} = f'(c)$
 (c) $\frac{f(b)-f(a)}{b-a} = f'(c)$ (d) $f'(c) = 0$
- j) The degree of the differential equation $\frac{d^2y}{dx^2} = \left[1 + \left(\frac{dy}{dx} \right)^3 \right]^{\frac{2}{3}}$ is _____. (01)
 (a) 1 (b) 2 (c) 3 (d) 6
- k) The equation $P(x, y)dx + Q(x, y)dy = 0$ is exact if _____. (01)
 (a) $P_x = Q_y$ (b) $P_y = Q_x$ (c) $P_x = -Q_y$ (d) $P_y = -Q_x$
- l) The solution of the differential equation $y + px = p^2$ is _____. (01)
 (a) $y = cx - c^2$ (b) $y = cx + \log c$ (c) $y = c^x + c^2$ (d) none of these
- m) What is the nth derivative of a^x ? (01)
- n) A $n \times n$ Non-Homogeneous system of equations $AX = B$ is given. If $\rho(A) = \rho(A : B) = n$ then the system has (01)
 (a) No solutions (b) Unique solutions
 (c) Infinite solution (d) None of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions

- a) Investigate for what value of λ and μ the equation (05)
 $x + 2y + z = 8; 2x + 2y + 2z = 13; 3x + 4y + \lambda z = \mu$ has
 i) unique solution, ii) infinite solution and iii) no solution.
- b) If $y = \sin^{-1} x$ then prove that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$. (05)
- c) Evaluate: $\lim_{x \rightarrow 0} (\cos x)^{\cot x}$ (04)



Q-3 Attempt all questions

- a) Solve the following system of linear equations (05)
 $x - y + z = 0; x + 2y + z = 0; 2x + y + 3z = 0$
- b) Solve: $(x^2 + y^2)dx + 2xy dy = 0; y(1) = 2$ (05)
- c) Expand $\log(1 + e^x)$ in ascending powers of x as far as the term containing x^4 . (04)

Q-4 Attempt all questions

- a) Evaluate: $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x \sin x}$ (05)
- b) Find n^{th} derivative of $\sin x \cdot \sin 2x$. (05)
- c) Find the centre and radius of the circle is given by (04)
 $x^2 + y^2 + z^2 + 12x - 12y - 16z + 111 = 0$ and $x + 2y + z = 17$.

Q-5 Attempt all questions

- a) Expand $\sin x$ in powers of $\left(x - \frac{\pi}{6}\right)$ by Taylor's theorem. (05)
- b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{bmatrix}$ by Gauss-Jordan method. (05)
- c) Find equation of sphere which passes through $(1, -3, 4), (1, -5, 2), (1, -3, 0)$ and whose centre lies on the plane $x + y + z = 0$. (04)

Q-6 Attempt all questions

- a) If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ then verify Cayley-Hamilton theorem and hence find A^4 . (07)
- b) Solve: $\frac{dy}{dx} + y = e^{2x}$ (03)
- c) State Rolle's theorem. Verify it and find value of c (04)
for $f(x) = e^x(\sin x - \cos x), \forall x \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

Q-7 Attempt all questions

- a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$. (07)



- b) If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ then find the characteristic polynomial of A and using it express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A (07)

Q-8 Attempt all questions

- a) State and prove Cayley-Hamilton theorem. (07)
- b) Solve: $p - \frac{1}{p} = \frac{x}{y} - \frac{y}{x}$ (04)
- c) Solve: $y = p(x-p) + \frac{a}{p}$ (03)

